

TOYAMA-92

Massless Limits of Massive Tensor Fields II

— *Infrared regularization of Fierz-Pauli model* —

Shinji HAMAMOTO *)

Department of Physics, Toyama University, Toyama 930

Abstract

Izawa's gauge-fixing procedure based on BRS symmetry is applied twice to the massive tensor field theory of Fierz-Pauli type. It is shown the second application can remove massless singularities which remain after the first application. Massless limit of the theory is discussed.

*) E-mail address: hamamoto@sci.toyama-u.ac.jp

§1. Introduction

In a previous paper¹⁾ (referred to as I), we investigated how Izawa's gauge-fixing procedure²⁾ based on BRS symmetry works for infrared regularization of massive tensor fields. We studied two models for a linearized massive tensor field, the pure-tensor (PT) type model by Fierz-Pauli and the additional-scalar-ghost (ASG) type one. It turned out that Izawa's procedure is effective for the ASG model, but not for the PT model. In the case of the ASG model, Izawa's procedure can regularize the original massless singularities of second order. On the other hand, the original singularities contained in the PT model are of fourth order, and Izawa's procedure can only reduce them to second order.

What we have learned from the above exercise is that when Izawa's procedure is applied once, massless singularities are reduced by second order. Now comes the question what happens when we apply Izawa's procedure once more to the PT model. This is the issue to be discussed in the present paper. We show that the second application of Izawa's procedure does regularize the remaining second-order massless singularities in the PT model.

In §2, the results obtained in I concerning the first application of Izawa's procedure are summarized. In §3, the second application of Izawa's procedure is performed to the PT model. It is shown that the second-order massless singularities which remain after the first application are in fact regularized. In §4, we discuss massless limit of the resulting theory. We see a graviton field is contained in the theory as a nonlocal combination of the basic fields. Summary and discussion are given in §5. In Appendix we show that applying Izawa's procedure to a massive vector field reproduces the Stueckelberg formalism.

§2. First application of Izawa's procedure

A massive tensor field is described by the Lagrangian^{*)}

$$L_h = \frac{1}{2}h^{\mu\nu}\Lambda_{\mu\nu,\rho\sigma}h^{\rho\sigma} - \frac{m^2}{2}(h^{\mu\nu}h_{\mu\nu} - ah^2), \quad (2.1)$$

where $\Lambda_{\mu\nu,\rho\sigma}$ is the operator defined by

$$\begin{aligned} \Lambda_{\mu\nu,\rho\sigma} = & (\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma})\square \\ & - (\eta_{\mu\rho}\partial_\nu\partial_\sigma + \eta_{\nu\sigma}\partial_\mu\partial_\rho) + (\eta_{\rho\sigma}\partial_\mu\partial_\nu + \eta_{\mu\nu}\partial_\rho\partial_\sigma), \end{aligned} \quad (2.2)$$

and a is a real parameter taking the values

$$a = \begin{cases} \frac{1}{2} & \text{for the ASG model,} \\ 1 & \text{for the PT model.} \end{cases} \quad (2.3)$$

^{*)} Notations used in this paper are the same as in I.

Two-point functions are calculated as

$$\begin{aligned}\langle h^{\mu\nu} h^{\rho\sigma} \rangle &= \frac{1}{\square - m^2} \left\{ \frac{1}{2} (\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma}) \right. \\ &\quad \left. - \frac{1}{2m^2} (\eta^{\mu\rho} \partial^\nu \partial^\sigma + \eta^{\mu\sigma} \partial^\nu \partial^\rho + \eta^{\nu\rho} \partial^\mu \partial^\sigma + \eta^{\nu\sigma} \partial^\mu \partial^\rho) \right\} \delta\end{aligned}\quad (2.4)$$

for the ASG model, and

$$\begin{aligned}\langle h^{\mu\nu} h^{\rho\sigma} \rangle &= \frac{1}{\square - m^2} \left\{ \frac{1}{2} (\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma}) \right. \\ &\quad \left. - \frac{1}{2m^2} (\eta^{\mu\rho} \partial^\nu \partial^\sigma + \eta^{\mu\sigma} \partial^\nu \partial^\rho + \eta^{\nu\rho} \partial^\mu \partial^\sigma + \eta^{\nu\sigma} \partial^\mu \partial^\rho) \right. \\ &\quad \left. + \frac{2}{3} \left(\frac{1}{2} \eta^{\mu\nu} + \frac{\partial^\mu \partial^\nu}{m^2} \right) \left(\frac{1}{2} \eta^{\rho\sigma} + \frac{\partial^\rho \partial^\sigma}{m^2} \right) \right\} \delta\end{aligned}\quad (2.5)$$

for the PT model. We see that the ASG model has second-order massless singularities, while the PT model fourth-order.

Applying Izawa's gauge-fixing procedure based on BRS symmetry, we obtain the following Lagrangian:

$$\begin{aligned}L_T &= \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} \\ &\quad - \frac{m^2}{2} \left[\left(h_{\mu\nu} - \frac{1}{m} (\partial_\mu \theta_\nu + \partial_\nu \theta_\mu) \right)^2 - a \left(h - \frac{2}{m} \partial^\mu \theta_\mu \right)^2 \right] \\ &\quad + b^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h + \frac{\alpha}{2} b_\mu \right) + i \bar{c}^\mu \square c_\mu,\end{aligned}\quad (2.6)$$

where an auxiliary vector field θ_μ , a Nakanishi-Lautrup (NL) field b_μ , a pair of Faddeev-Popov (FP) ghosts (c_μ, \bar{c}_μ) , and a gauge parameter α have been introduced. This Lagrangian is invariant under the following BRS transformation:

$$\delta h_{\mu\nu} = \partial_\mu c_\nu + \partial_\nu c_\mu, \quad \delta \theta_\mu = m c_\mu, \quad \delta \bar{c}_\mu = i b_\mu. \quad (2.7)$$

Putting $a = \frac{1}{2}$ in Eq.(2.5), we have for the ASG model

$$\begin{aligned}L_{T,a=\frac{1}{2}} &= \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} \\ &\quad - \frac{m^2}{2} \left(h^{\mu\nu} h_{\mu\nu} - \frac{1}{2} h^2 \right) - 2m \theta^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h \right) - \partial_\mu \theta_\nu \partial^\mu \theta^\nu \\ &\quad + b^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h + \frac{\alpha}{2} b_\mu \right) + i \bar{c}^\mu \square c_\mu.\end{aligned}\quad (2.8)$$

This gives the following two-point functions:

$$\begin{aligned} \langle h^{\mu\nu} h^{\rho\sigma} \rangle &= \frac{1}{\square - m^2} \left\{ \frac{1}{2} (\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma}) \right. \\ &\quad \left. - \frac{1}{2} \left[(1 - 2\alpha) \frac{1}{\square} + 2\alpha \frac{m^2}{\square^2} \right] (\eta^{\mu\rho} \partial^\nu \partial^\sigma + \eta^{\mu\sigma} \partial^\nu \partial^\rho + \eta^{\nu\rho} \partial^\mu \partial^\sigma + \eta^{\nu\sigma} \partial^\mu \partial^\rho) \right\} \delta, \end{aligned} \quad (2.9)$$

$$\langle h^{\mu\nu} b^\rho \rangle = \frac{1}{\square} (\eta^{\mu\rho} \partial^\nu + \eta^{\nu\rho} \partial^\mu) \delta, \quad (2.10)$$

$$\langle h^{\mu\nu} \theta^\rho \rangle = -\alpha m \frac{1}{\square^2} (\eta^{\mu\rho} \partial^\nu + \eta^{\nu\rho} \partial^\mu) \delta, \quad (2.11)$$

$$\langle b^\mu b^\rho \rangle = 0, \quad (2.12)$$

$$\langle b^\mu \theta^\rho \rangle = m \frac{1}{\square} \eta^{\mu\rho} \delta, \quad (2.13)$$

$$\langle \theta^\mu \theta^\rho \rangle = \frac{1}{2} \frac{1}{\square} \left(1 - 2\alpha \frac{m^2}{\square} \right) \eta^{\mu\rho} \delta, \quad (2.14)$$

except for the trivial one for $\langle c^\mu \bar{c}^\nu \rangle$. *) We see that the massless singularities in (2.4) have been regularized by this procedure. **)

When $a = 1$, the Lagrangian (2.6) gives

$$\begin{aligned} L_{T,a=1} &= \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} \\ &\quad - \frac{m^2}{2} (h^{\mu\nu} h_{\mu\nu} - h^2) - 2m\theta^\mu (\partial^\nu h_{\mu\nu} - \partial_\mu h) - \frac{1}{2} (\partial_\mu \theta_\nu - \partial_\nu \theta_\mu)^2 \\ &\quad + b^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h + \frac{\alpha}{2} b_\mu \right) + i\bar{c}^\mu \square c_\mu. \end{aligned} \quad (2.15)$$

Thus two-point functions for the PT model are

$$\begin{aligned} \langle h^{\mu\nu} h^{\rho\sigma} \rangle &= \frac{1}{\square - m^2} \left\{ \frac{1}{2} (\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma}) \right. \\ &\quad \left. - \frac{1}{2} \left[(1 - 2\alpha) \frac{1}{\square} + 2\alpha \frac{m^2}{\square^2} \right] (\eta^{\mu\rho} \partial^\nu \partial^\sigma + \eta^{\mu\sigma} \partial^\nu \partial^\rho + \eta^{\nu\rho} \partial^\mu \partial^\sigma + \eta^{\nu\sigma} \partial^\mu \partial^\rho) \right. \\ &\quad \left. + \frac{2}{3} \left(\frac{1}{2} \eta^{\mu\nu} + \frac{\partial^\mu \partial^\nu}{\square} \right) \left(\frac{1}{2} \eta^{\rho\sigma} + \frac{\partial^\rho \partial^\sigma}{\square} \right) \right\} \delta, \end{aligned} \quad (2.16)$$

$$\langle h^{\mu\nu} b^\rho \rangle = \frac{1}{\square} (\eta^{\mu\rho} \partial^\nu + \eta^{\nu\rho} \partial^\mu) \delta, \quad (2.17)$$

$$\langle h^{\mu\nu} \theta^\rho \rangle = \left\{ \frac{1}{6m} \frac{1}{\square} \eta^{\mu\nu} \partial^\rho - \alpha m \frac{1}{\square^2} (\eta^{\mu\rho} \partial^\nu + \eta^{\nu\rho} \partial^\mu) + \frac{1}{3m} \frac{1}{\square^2} \partial^\mu \partial^\nu \partial^\rho \right\} \delta, \quad (2.18)$$

*) In Abelian cases which we are dealing with, FP ghosts decouple from all other fields to give trivial two-point functions. We omit to write down their explicit forms throughout the paper.

**) Exactly speaking, the quantity \square^{-2} is well-defined only when accompanied by derivatives. Therefore, the expression (2.14) is meaningful only in a formal sense. If the field θ_μ has some non-derivative couplings, α is to be set 0. On the other hand, if it has derivative couplings only, then any value of α is allowed.

$$\langle b^\mu b^\rho \rangle = 0, \quad (2.19)$$

$$\langle b^\mu \theta^\rho \rangle = m \frac{1}{\square} \eta^{\mu\rho} \delta, \quad (2.20)$$

$$\langle \theta^\mu \theta^\rho \rangle = \left\{ \frac{1}{2} \frac{1}{\square} \left(1 - 2\alpha \frac{m^2}{\square} \right) \eta^{\mu\rho} - \frac{1}{6m^2} \frac{1}{\square} \left(1 - \frac{m^2}{\square} \right) \partial^\mu \partial^\rho \right\} \delta. \quad (2.21)$$

We see the fourth-order massless singularities found in (2.5) for the original PT model have been driven away. However, new at-most-second-order singularities have appeared in the θ -sector (2.18) and (2.21). Can these singularities be driven away by applying Izawa's procedure once more? This is the issue to address in the next section.

§3. Second application of Izawa's procedure

The starting point is the Lagrangian (2.15). The kinetic term of the vector field θ_μ shows this field is a kind of gauge field. It is expected from this fact that the second application of Izawa's procedure works. This is in fact the case as seen below.

We introduce a new set of variables (θ'_μ, φ) to perform a field transformation $\theta \rightarrow (\theta'_\mu, \varphi)$ such that

$$\theta_\mu = \theta'_\mu - \frac{1}{m} \partial_\mu \varphi, \quad (3.1)$$

$$\partial^\mu \theta'_\mu = 0. \quad (3.2)$$

The new variables (θ'_μ, φ) are first assumed to be independent of the old one θ_μ . Then the Lagrangian (2.15), which does not depend on the new variables, is invariant under the BRS transformation

$$\begin{cases} \delta' \theta'_\mu = c'_\mu, & \delta' \bar{c}'_\mu = i b'_\mu, \\ \delta' \varphi = mc, & \delta' \bar{c} = ib, \end{cases} \quad (3.3)$$

where the new FP ghosts (c'_μ, c) and (\bar{c}'_μ, \bar{c}) as well as the new NL fields (b'_μ, b) have been introduced. To relate the old and new sets of variables, we supplement the Lagrangian (2.15) by adding the following BRS gauge-fixing term:

$$\begin{aligned} L'_B &= -i\delta' \left[\bar{c}^\mu \left(\theta_\mu - \theta'_\mu + \frac{1}{m} \partial_\mu \varphi \right) + \bar{c} \left(\partial^\mu \theta'_\mu - \frac{m}{2} h + \frac{\beta}{2} b \right) \right] \\ &= b^\mu \left(\theta_\mu - \theta'_\mu + \frac{1}{m} \partial_\mu \varphi \right) + b \left(\partial^\mu \theta'_\mu - \frac{m}{2} h + \frac{\beta}{2} b \right) \\ &\quad - i (\bar{c}^\mu + \partial^\mu \bar{c}) (c'_\mu - \partial_\mu c) + i \bar{c} \square c \end{aligned} \quad (3.4)$$

with the second gauge parameter β . The path integral is given as

$$\begin{aligned} Z &= \int \mathcal{D}h_{\mu\nu} \mathcal{D}\theta_\mu \mathcal{D}b_\mu \mathcal{D}c_\mu \mathcal{D}\bar{c}_\mu \mathcal{D}\theta'_\mu \mathcal{D}\varphi \mathcal{D}b'_\mu \mathcal{D}c'_\mu \mathcal{D}\bar{c}'_\mu \mathcal{D}b \mathcal{D}c \mathcal{D}\bar{c} \\ &\quad \times \exp i \int d^4x [L_{T,a=1} + L'_B]. \end{aligned} \quad (3.5)$$

Integrating over the variables $(b'_\mu, \theta_\mu, c'_\mu, \bar{c}'_\mu)$ and overwriting θ_μ on θ'_μ , we obtain

$$Z = \int \mathcal{D}h_{\mu\nu} \mathcal{D}\theta_\mu \mathcal{D}\varphi \mathcal{D}b_\mu \mathcal{D}b \mathcal{D}c_\mu \mathcal{D}\bar{c}_\mu \mathcal{D}c \mathcal{D}\bar{c} \exp i \int d^4x L'_{T,a=1}, \quad (3.6)$$

where

$$\begin{aligned} L'_{T,a=1} = & \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} \\ & - \frac{m^2}{2} \left[\left(h_{\mu\nu} - \frac{1}{m} (\partial_\mu \theta_\nu + \partial_\nu \theta_\mu) + \frac{2}{m^2} \partial_\mu \partial_\nu \varphi \right)^2 \right. \\ & \quad \left. - \left(h - \frac{2}{m} \partial^\mu \theta_\mu + \frac{2}{m^2} \square \varphi \right)^2 \right] \\ & + b^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h + \frac{\alpha}{2} b_\mu \right) + i \bar{c}^\mu \square c_\mu \\ & + b \left(\partial^\mu \theta_\mu - \frac{m}{2} h + \frac{\beta}{2} b \right) + i \bar{c} \square c \\ = & \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} - \frac{m^2}{2} \left(h^{\mu\nu} h_{\mu\nu} - h^2 \right) \\ & - 2(m\theta^\mu - \partial^\mu \varphi) (\partial^\nu h_{\mu\nu} - \partial_\mu h) - \frac{1}{2} (\partial_\mu \theta_\nu - \partial_\nu \theta_\mu)^2 \\ & + b^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h + \frac{\alpha}{2} b_\mu \right) + i \bar{c}^\mu \square c_\mu \\ & + b \left(\partial^\mu \theta_\mu - \frac{m}{2} h + \frac{\beta}{2} b \right) + i \bar{c} \square c. \end{aligned} \quad (3.7)$$

This Lagrangian is invariant under the following BRS transformation:

$$\begin{cases} \delta h_{\mu\nu} = \partial_\mu c_\nu + \partial_\nu c_\mu, & \delta \bar{c}_\mu = i b_\mu, \\ \delta \theta_\mu = m c_\mu + \partial_\mu c, & \delta \bar{c} = i b, \\ \delta \varphi = m c. \end{cases} \quad (3.8)$$

We note the Lagrangian $L'_{T,a=1}$ has a smooth massless limit. This comes from the fact that the kinetic term of θ_μ contained in the Lagrangian $L_{T,a=1}$ (2.15) is the gauge-theoretic one $-\frac{1}{2} (\partial_\mu \theta_\nu - \partial_\nu \theta_\mu)^2$. On the contrary, the Lagrangian $L_{T,a=\frac{1}{2}}$ (2.8) for the ASG model has the non-gauge-theoretic kinetic term $-\partial_\mu \theta_\nu \partial^\mu \theta^\nu$. When performed the second application of Izawa's procedure, such a term yields at-most-second-order singular terms like $-\frac{1}{m^2} (\square \varphi)^2$.

Two-point functions obtained from $L'_{T,a=1}$ (3.7) are the following:

$$\begin{aligned} \langle h^{\mu\nu} h^{\rho\sigma} \rangle = & \frac{1}{\square - m^2} \left\{ \frac{1}{2} (\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma}) \right. \\ & \left. - \frac{1}{2} \left[(1 - 2\alpha) \frac{1}{\square} + 2\alpha \frac{m^2}{\square^2} \right] (\eta^{\mu\rho} \partial^\nu \partial^\sigma + \eta^{\mu\sigma} \partial^\nu \partial^\rho + \eta^{\nu\rho} \partial^\mu \partial^\sigma + \eta^{\nu\sigma} \partial^\mu \partial^\rho) \right\} \end{aligned}$$

$$+ \frac{2}{3} \left(\frac{1}{2} \eta^{\mu\nu} + \frac{\partial^\mu \partial^\nu}{\square} \right) \left(\frac{1}{2} \eta^{\rho\sigma} + \frac{\partial^\rho \partial^\sigma}{\square} \right) \} \delta, \quad (3.9)$$

$$\langle h^{\mu\nu} b^\rho \rangle = \frac{1}{\square} (\eta^{\mu\rho} \partial^\nu + \eta^{\nu\rho} \partial^\mu) \delta, \quad (3.10)$$

$$\langle h^{\mu\nu} b \rangle = 0, \quad (3.11)$$

$$\langle h^{\mu\nu} \theta^\rho \rangle = - \alpha m \frac{1}{\square^2} (\eta^{\mu\rho} \partial^\nu + \eta^{\nu\rho} \partial^\mu) \delta, \quad (3.12)$$

$$\langle h^{\mu\nu} \varphi \rangle = \frac{1}{3} \frac{1}{\square} \left(\frac{1}{2} \eta^{\mu\nu} + \frac{\partial^\mu \partial^\nu}{\square} \right) \delta, \quad (3.13)$$

$$\langle b^\mu b^\rho \rangle = 0, \quad (3.14)$$

$$\langle b^\mu b \rangle = 0, \quad (3.15)$$

$$\langle b^\mu \theta^\rho \rangle = m \frac{1}{\square} \eta^{\mu\rho} \delta, \quad (3.16)$$

$$\langle b^\mu \varphi \rangle = 0, \quad (3.17)$$

$$\langle bb \rangle = 0, \quad (3.18)$$

$$\langle b \theta^\mu \rangle = - \frac{1}{\square} \partial^\mu \delta, \quad (3.19)$$

$$\langle b \varphi \rangle = m \frac{1}{\square} \delta, \quad (3.20)$$

$$\langle \theta^\mu \theta^\rho \rangle = \left[\frac{1}{2} \frac{1}{\square} \left(1 - 2\alpha \frac{m^2}{\square} \right) \eta^{\mu\rho} - \frac{1}{2} (1 - 2\beta) \frac{\partial^\mu \partial^\rho}{\square^2} \right] \delta, \quad (3.21)$$

$$\langle \theta^\mu \varphi \rangle = \frac{1}{2} (1 - 2\beta) \frac{m}{\square^2} \partial^\mu \delta, \quad (3.22)$$

$$\langle \varphi \varphi \rangle = \left[\frac{1}{6} \frac{1}{\square} \left(1 - \frac{m^2}{\square} \right) + \frac{1}{2} (1 - 2\beta) \frac{m^2}{\square^2} \right] \delta. \quad (3.23)$$

These expressions show that the massless singularities remaining in (2.18) and (2.21) have been regularized. *) Izawa's procedure does work in this case.

§4. Massless limit

It has been found that the theory constructed in the previous section has a smooth massless limit. In this section, we investigate whether or not the limit is consistent with the ordinary massless tensor theory.

As m tends to 0, the Lagrangian $L'_{T,a=1}$ (3.7) reduces to

$$L = \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} + 2\partial^\mu \varphi (\partial^\nu h_{\mu\nu} - \partial_\mu h) + b^\mu \left(\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h + \frac{\alpha}{2} b_\mu \right)$$

*) The note stated in the second footnote on p. 4 holds here too. For $\langle \theta^\mu \theta^\rho \rangle$ in (3.21) to be well-defined, either the gauge parameter α should be set 0 or the field θ_μ should appear in company with derivatives in interaction Lagrangian. For $\langle \varphi \varphi \rangle$ in (3.23) to be well-defined, either the gauge parameter β should be chosen as $\frac{1}{3}$ or the field φ should be accompanied by derivatives in interaction Lagrangian.

$$-\frac{1}{2}(\partial_\mu\theta_\nu - \partial_\nu\theta_\mu)^2 + b\left(\partial^\mu\theta_\mu + \frac{\beta}{2}b\right), \quad (4.1)$$

where the trivial FP-ghost terms have been omitted. The $(h_{\mu\nu}, \varphi, b_\mu)$ -sector is completely separated from the (θ_μ, b) -sector. If it were not for the second term in the first line of the right hand side of (4.1), the $(h_{\mu\nu}, \varphi, b_\mu)$ -sector coincides with the ordinary massless tensor theory. Two-point functions are for the $(h_{\mu\nu}, \varphi, b_\mu)$ -sector

$$\begin{aligned} \langle h^{\mu\nu}h^{\rho\sigma} \rangle &= \frac{1}{\square} \left\{ \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}) \right. \\ &\quad - \frac{1}{2}(1-2\alpha)\frac{1}{\square}(\eta^{\mu\rho}\partial^\nu\partial^\sigma + \eta^{\mu\sigma}\partial^\nu\partial^\rho + \eta^{\nu\rho}\partial^\mu\partial^\sigma + \eta^{\nu\sigma}\partial^\mu\partial^\rho) \\ &\quad \left. + \frac{2}{3}\left(\frac{1}{2}\eta^{\mu\nu} + \frac{\partial^\mu\partial^\nu}{\square}\right)\left(\frac{1}{2}\eta^{\rho\sigma} + \frac{\partial^\rho\partial^\sigma}{\square}\right)\right\} \delta, \end{aligned} \quad (4.2)$$

$$\langle h^{\mu\nu}b^\rho \rangle = \frac{1}{\square}(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu)\delta, \quad (4.3)$$

$$\langle h^{\mu\nu}\varphi \rangle = \frac{1}{3}\frac{1}{\square}\left(\frac{1}{2}\eta^{\mu\nu} + \frac{\partial^\mu\partial^\nu}{\square}\right)\delta, \quad (4.4)$$

$$\langle b^\mu b^\rho \rangle = 0, \quad (4.5)$$

$$\langle b^\mu\varphi \rangle = 0, \quad (4.6)$$

$$\langle \varphi\varphi \rangle = \frac{1}{6}\frac{1}{\square}\delta, \quad (4.7)$$

and for the (θ_μ, b) -sector

$$\langle \theta^\mu\theta^\rho \rangle = \frac{1}{2}\frac{1}{\square}\left\{\eta^{\mu\rho} - (1-2\beta)\frac{\partial^\mu\partial^\rho}{\square}\right\}\delta, \quad (4.8)$$

$$\langle \theta^\mu b \rangle = \frac{1}{\square}\partial^\mu\delta, \quad (4.9)$$

$$\langle bb \rangle = 0. \quad (4.10)$$

The $(h_{\mu\nu}, \varphi, b_\mu)$ -sector does not reproduce the two-point functions for the ordinary massless tensor theory. If we didn't have the third line in the expression (4.2) for $\langle h^{\mu\nu}h^{\rho\sigma} \rangle$, and if $\langle h^{\mu\nu}\varphi \rangle$ were 0 in (4.4), then the whole set of two-point functions agrees with that of the ordinary massless tensor.

In order to see how the ordinary graviton field is contained in this model, we now introduce the following nonlocal combination of the basic fields:

$$H_{\mu\nu} = h_{\mu\nu} - 2\left(\frac{1}{2}\eta_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{\square}\right)\varphi. \quad (4.11)$$

For this new field we have

$$\langle H^{\mu\nu}H^{\rho\sigma} \rangle = \frac{1}{\square}\left\{\frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})\right.$$

$$-\frac{1}{2}(1-2\alpha)\frac{1}{\square}(\eta^{\mu\rho}\partial^\nu\partial^\sigma + \eta^{\mu\sigma}\partial^\nu\partial^\rho + \eta^{\nu\rho}\partial^\mu\partial^\sigma + \eta^{\nu\sigma}\partial^\mu\partial^\rho)\Big\}\delta, \quad (4.12)$$

$$\langle H^{\mu\nu}b^\rho \rangle = \frac{1}{\square}(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu)\delta, \quad (4.13)$$

$$\langle H^{\mu\nu}\varphi \rangle = 0 \quad (4.14)$$

$$(4.15)$$

instead of (4.2), (4.3) and (4.4). Thus the two-point functions for the ordinary massless tensor field are in fact provided by the field $H_{\mu\nu}$.

§5. Summary and discussion

It has turned out that:

- (1) The original ASG model for a massive tensor field has second-order massless singularities, which can be regularized by applying Izawa's procedure based on BRS symmetry once;
- (2) The original PT model develops fourth-order massless singularities, which can be regularized by applying Izawa's procedure twice.

The Batalin-Fradkin algorithm³⁾ is another powerful method for constructing gauge-invariant theories from non-gauge-invariant ones. The application of this procedure to a massive tensor field has been performed in Ref. 4), giving the same Lagrangians as obtained above, $L_{T,a=\frac{1}{2}}$ (2.8) for the ASG model and $L'_{T,a=1}$ (3.7) for the PT model.

We now have massive tensor theories equipped with BRS invariance as well as smooth massless limits. However, we are still in the linearized world. To construct complete nonlinear theories is a major problem to solve.

Acknowledgements

We would like to thank T. Kurimoto for discussion.

Appendix

— *On the Stueckelberg Formalism* —

It is known a massive vector field described by

$$L_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu \quad (A.1)$$

develops massless singularities in the limit $m = 0$. There are some methods to regularize such singularities. The most popular one is the Stueckelberg formalism.⁵⁾ In this formalism an additional scalar field B is introduced and the Lagrangian and the physical state condition are given as

$$L_S = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu - \frac{m^2}{2}A_\mu A^\mu - \frac{1}{2}\partial_\mu B \partial^\mu B - \frac{m^2}{2}B^2, \quad (\text{A.2})$$

$$(\partial^\mu A_\mu + mB)^{(+)} | \rangle = 0. \quad (\text{A.3})$$

On the other hand, as shown in I, Izawa's procedure based on BRS symmetry can also give a massless-regular theory, in which the Lagrangian is given by ^{*)}

$$L_{\text{BRS}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}\left(A_\mu - \frac{1}{m}\partial_\mu\theta\right)^2 + b\left(\partial^\mu A_\mu + \frac{\alpha}{2}b\right) + i\bar{c}\square c \quad (\text{A.4})$$

with an auxiliary scalar field θ , an NL field b , a pair of FP ghosts (c, \bar{c}) , and a gauge parameter α . This Appendix is devoted to show these two formulations are equivalent with each other. To do that we have to establish both the equivalence of path integrals and that of physical state conditions.

The path integral of the BRS-Izawa theory is

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\theta \mathcal{D}b \mathcal{D}c \mathcal{D}\bar{c} \exp i \int d^4x L_{\text{BRS}}. \quad (\text{A.5})$$

This is equivalent to

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\theta \delta(\partial^\mu A_\mu - f) \text{Det}N \exp i \int d^4x L'_{\text{BRS}}, \quad (\text{A.6})$$

where L'_{BRS} and N are defined by

$$\begin{aligned} L'_{\text{BRS}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}\left(A_\mu - \frac{1}{m}\partial_\mu\theta\right)^2 \\ &= -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu - \frac{m^2}{2}A_\mu A^\mu - \frac{1}{2}\partial_\mu\theta\partial^\mu\theta - \frac{m^2}{2}\theta^2 \\ &\quad + \frac{1}{2}(\partial^\mu A_\mu - m\theta)^2, \end{aligned} \quad (\text{A.7})$$

$$N = \square\delta^4(x - x'), \quad (\text{A.8})$$

and f is an arbitrary function of x . The expression (A.6) corresponds to the gauge-fixing condition

$$\partial^\mu A_\mu = f. \quad (\text{A.9})$$

We are allowed to take another gauge fixing condition

$$\partial^\mu A_\mu - m\theta = f' \quad (\text{A.10})$$

^{*)} The same Lagrangian is also obtained by applying the Batalin-Fradkin algorithm.⁶⁾

with another arbitrary function f' . Because the Lagrangian L'_{BRS} is invariant under the gauge transformation

$$\delta A_\mu = \partial_\mu \varepsilon, \quad \delta \theta = m \varepsilon, \quad (\text{A.11})$$

the factor

$$\delta(\partial^\mu A_\mu - f) \text{Det}N \quad (\text{A.12})$$

in (A.6) can be replaced by

$$\delta(\partial^\mu A_\mu - m\theta - f') \text{Det}N' \quad (\text{A.13})$$

with

$$N' = (\square - m^2) \delta^4(x - x'). \quad (\text{A.14})$$

That is

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\theta \delta(\partial^\mu A_\mu - m\theta - f') \text{Det}N' \exp i \int d^4x L'_{\text{BRS}}. \quad (\text{A.15})$$

Taking into account the f' -independence for Z , we multiply (A.15) by

$$1 = \int \mathcal{D}f' \exp i \int d^4x \left(-\frac{1}{2} f'^2 \right). \quad (\text{A.16})$$

Then we have

$$\begin{aligned} Z &= \int \mathcal{D}A_\mu \mathcal{D}\theta \mathcal{D}f' \delta(\partial^\mu A_\mu - m\theta - f') \text{Det}N' \exp i \int d^4x \left(L'_{\text{BRS}} - \frac{1}{2} f'^2 \right) \\ &= \int \mathcal{D}A_\mu \mathcal{D}\theta \text{Det}N' \exp i \int d^4x L''_{\text{BRS}}, \end{aligned} \quad (\text{A.17})$$

where

$$L''_{\text{BRS}} = -\frac{1}{2} \partial_\mu A_\nu \partial^\mu A^\nu - \frac{m^2}{2} A_\mu A^\mu - \frac{1}{2} \partial_\mu \theta \partial^\mu \theta - \frac{m^2}{2} \theta^2. \quad (\text{A.18})$$

This Lagrangian L''_{BRS} shares the same form with the Stueckelberg Lagrangian L_S (A.2). The equivalence of the path integrals has thus been confirmed.

Next come the physical state conditions. The expression (A.15) is equivalent to

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\theta \mathcal{D}b \mathcal{D}c \mathcal{D}\bar{c} \exp i \int d^4x L'''_{\text{BRS}}, \quad (\text{A.19})$$

where

$$\begin{aligned} L'''_{\text{BRS}} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2} \left(A_\mu - \frac{1}{m} \partial_\mu \theta \right)^2 \\ &\quad + b \left(\partial^\mu A_\mu - m\theta + \frac{\alpha}{2} b \right) + i\bar{c} (\square - m^2) c. \end{aligned} \quad (\text{A.20})$$

Because this Lagrangian is invariant under the BRS transformation

$$\delta A_\mu = \partial_\mu c, \quad \delta \theta = mc, \quad \delta \bar{c} = ib, \quad (\text{A.21})$$

a conserved BRS charge Q_B is defined. This is expressed as

$$Q_B = \int d^3x (b \partial_0 c - \partial_0 b \cdot c). \quad (\text{A.22})$$

By the use of this charge, we can impose as usual the physical state condition

$$Q_B | \rangle = 0, \quad (\text{A.23})$$

which is shown to be equivalent to the condition

$$b^{(+)}(x) | \rangle = 0 \quad \text{for } \forall x. \quad (\text{A.24})$$

The field equation

$$\partial^\mu A_\mu - m\theta + \alpha b = 0 \quad (\text{A.25})$$

tells the condition (A.24) reduces to

$$(\partial^\mu A_\mu(x) - m\theta(x))^{(+)} | \rangle = 0 \quad \text{for } \forall x. \quad (\text{A.26})$$

This agrees with the condition (A.3). The equivalence of the physical state conditions has thus been confirmed too.

References

- [1] S. Hamamoto, Prog. Theor. Phys. **96** (1996), 639.
- [2] K.-I. Izawa, Prog. Theor. Phys. **88** (1992), 759.
- [3] I. A. Batalin and E. S. Fradkin, Phys. Lett. **B180** (1986), 157; Nucl. Phys. **B279** (1987), 514.
- [4] S. Hamamoto, Preprint TOYAMA-91, hep-th/9611021.
- [5] E. C. G. Stueckelberg, Helv. Phys. Acta **11** (1938), 225, 299.
See also N. Nakanishi, Prog. Theor. Phys. Suppl. No.51 (1972), 1.
- [6] Y. Igarashi, Soryushiron Kenkyu (Kyoto) **82** (1991), 414.
See also S. Hamamoto, Prog. Theor. Phys. **95** (1996), 441.